The nature of instabilities in blocked media and seismological law of Gutenberg-Richter

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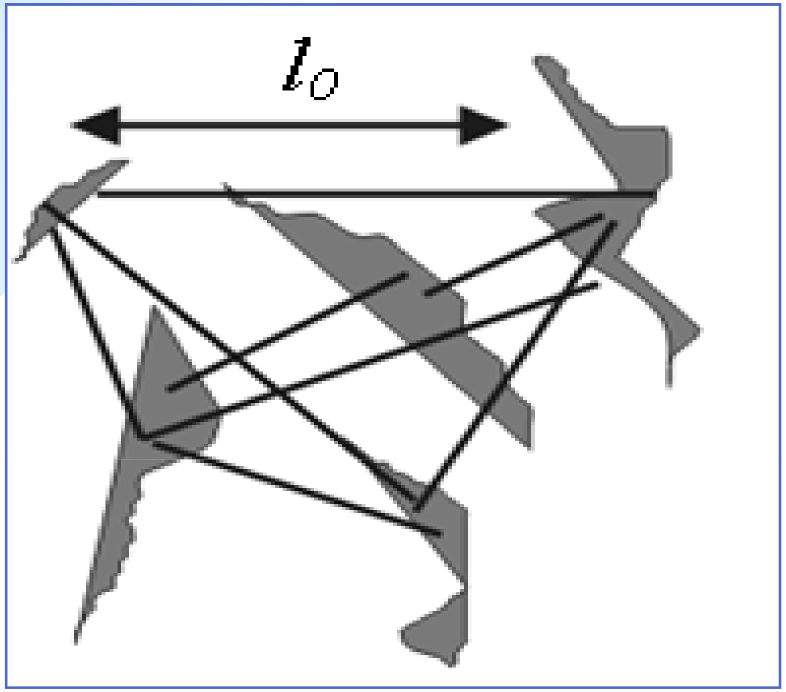
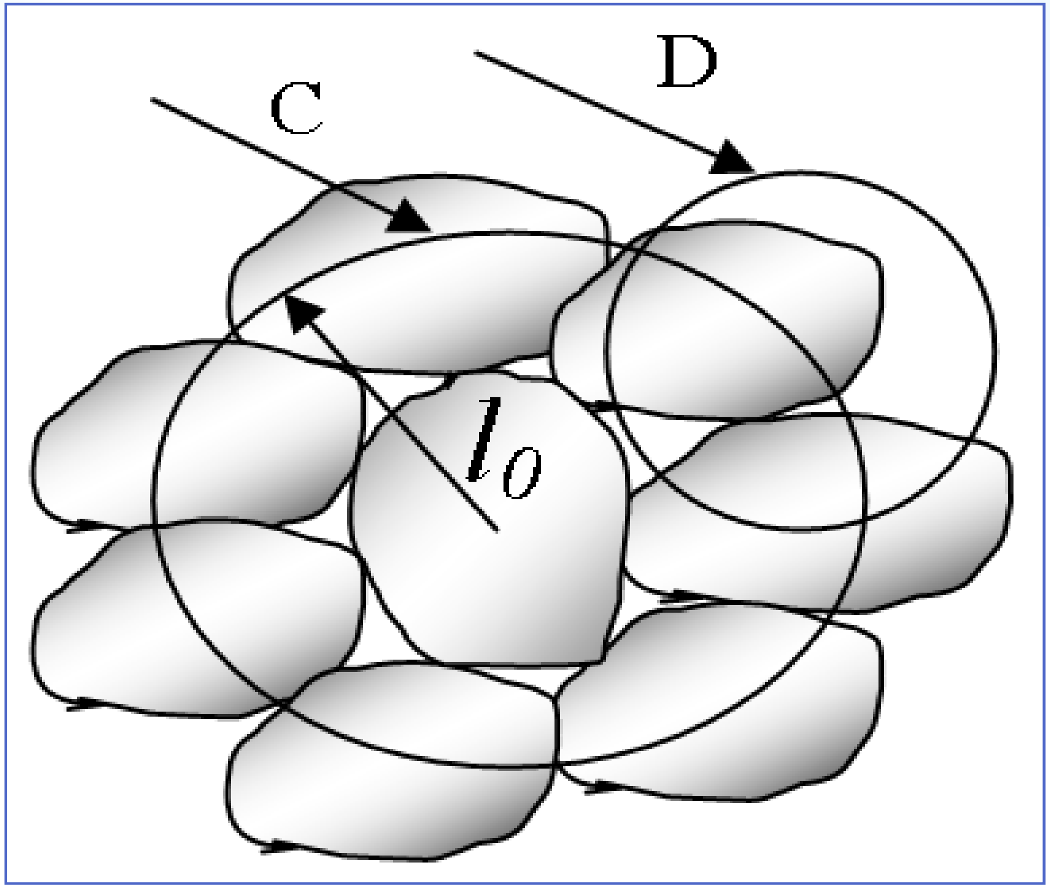
Two alternatives. 1. Classic continuum with boundary conditions on the internal surfaces. 2. Another continuum model, where integral geometry of pore space there is in origin.



$$\sigma_0 l_0 = 4(1 - f)$$

The element of a structured body with an average distance l_0 **between pores.**

The problem of creation of equilibrium equation into arbitrary element of discrete medium



$$u(x \pm l_0) = u(x) e^{\pm l_0 D_x} \qquad D_x = \frac{\partial}{\partial x}$$

The translated forces is fieling all space including pores and cracks

The difference of the first order is given by

$$\Delta_1 = u(x)\frac{1}{l_0}\left(e^{\frac{l_0 D_x}{2}} - e^{-\frac{l_0 D_x}{2}}\right) = u(x)\frac{\sinh\left(\frac{l_0 D_x}{2}\right)}{\frac{l_0}{2}}$$

Expression tends to the first derivative at $l_0 \to 0$. Analogously the second difference may be written as

$$\Delta_2 = u(x)\frac{1}{l_0^2}\left(e^{\frac{l_0 D_x}{2}} - e^{-\frac{l_0 D_x}{2}}\right)^2 = u(x)\frac{\sinh^2\left(\frac{l_0 D_x}{2}\right)}{\left(\frac{l_0}{2}\right)^2}$$

Expression in (5) tends to the second derivative at $l_0 \to 0$. The similar operator of translation in three-dimension space for some sphere is given by expression

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Expression tends to the second derivative at $l_0 \to 0$. The similar operator of translation in three-dimension space for some sphere is given by expression

$$P(D_x, D_y, D_z, l_0) =$$

$$= \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \exp[l_0 (D_x \sin\theta \cos\varphi +$$

$$+ D_y \sin\theta \sin\varphi + D_z \cos\theta)] \sin\theta d\theta d\varphi =$$

$$= \frac{\sinh(l_0 \sqrt{\Delta})}{l_0 \sqrt{\Delta}} = E + \frac{l_0^2}{3!} \Delta + \frac{l_0^4}{5!} \Delta\Delta + \ldots.$$

According to Poisson formula we have

$$\int_0^{2\pi} \int_0^{\pi} f(\alpha \cos\theta + \beta \sin\theta Cos\varphi +$$

$$+ \gamma \sin\theta \sin\varphi) \sin\theta d\theta d\varphi =$$

$$= 2\pi \int_0^{\pi} f(\sqrt{\alpha^2 + \beta^2 + \gamma^2} \cos p) \sin p dp$$

so the operator *P* may be rewritten as follows

$$P(D_x, D_y, D_z; l_0) = \frac{1}{2}\int_{-1}^{1} \exp(l_0\sqrt{\Delta}\, t)dt =$$

$$= \int_{0}^{1} \cosh(l_0\sqrt{\Delta}\, t)dt = \frac{\sinh(l_0\sqrt{\Delta})}{l_0\sqrt{\Delta}} =$$

$$= E + \frac{l_0^2\Delta}{3!} + \frac{l_0^4\Delta\Delta}{5!} + \ldots$$

P=E corresponds to classic continuum model by Cauchy and Poisson

The equation of motion in blocked media

$$\frac{\partial}{\partial x_k}[P(\sigma_{ik})] = \rho\ddot{u}_i \qquad \frac{\partial}{\partial x_k}\left(E + \frac{l_0^2}{3!}\Delta + \frac{l_0^4}{5!}\Delta\Delta + \ldots\right)\sigma_{ik} = \rho\ddot{u}_i$$

For one dimensional case equation takes more simple expression:

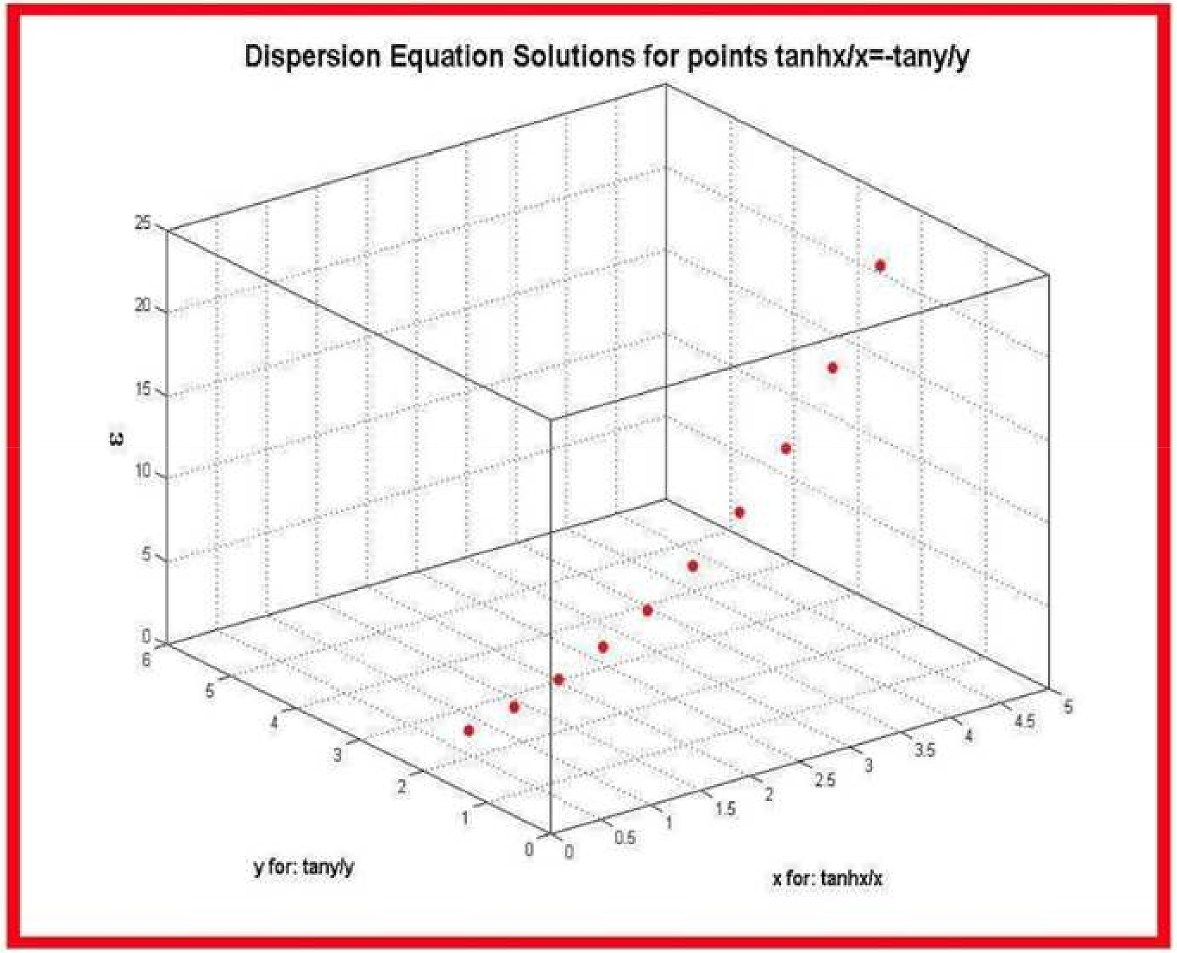
$$u''(E+\frac{l_0^2\Delta}{3!}+\frac{l_0^4\Delta\Delta}{5!}+....)+k_S^2u=0 \qquad u=Aexp\left(ikx\right)$$

$$\frac{\sin(kl_0)}{kl_0}-\frac{k_S^2}{k^2}=0$$

Or, for unknown wave velocity, which depends on range of structure or specific surface of sample. In the formula *Ks* is usual wave number for shear or longitude waves.

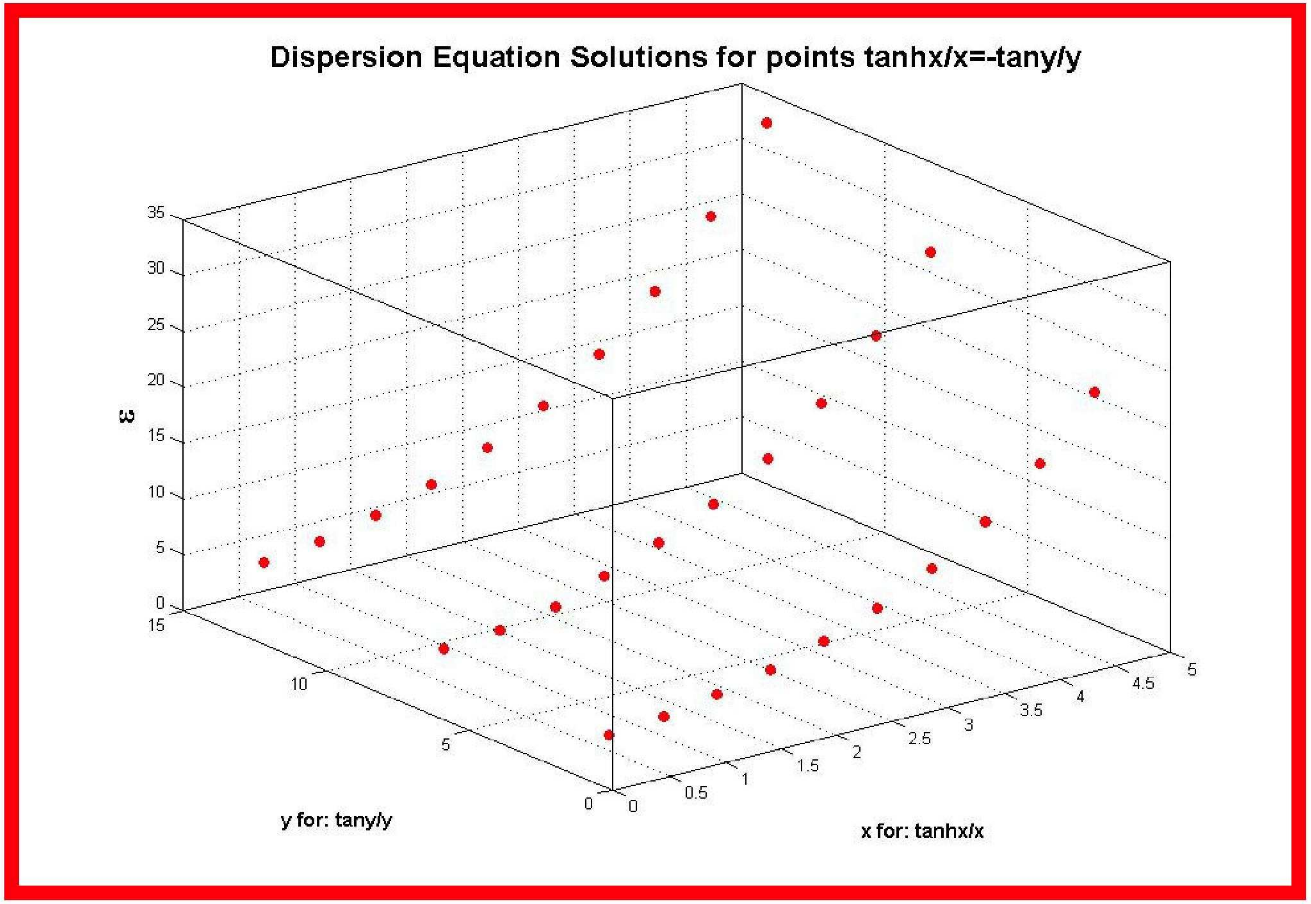
In the case of $l_0 \to 0$ the wave number $k=k_s$, i.e. the wave velocity is equal to V_s or V_p elastic wave velocities. However, if l_0 is not very small value, the wave velocity decreases up to a zero when $kl_0 = \pi m$, where *m* is an integer number.

Small scale



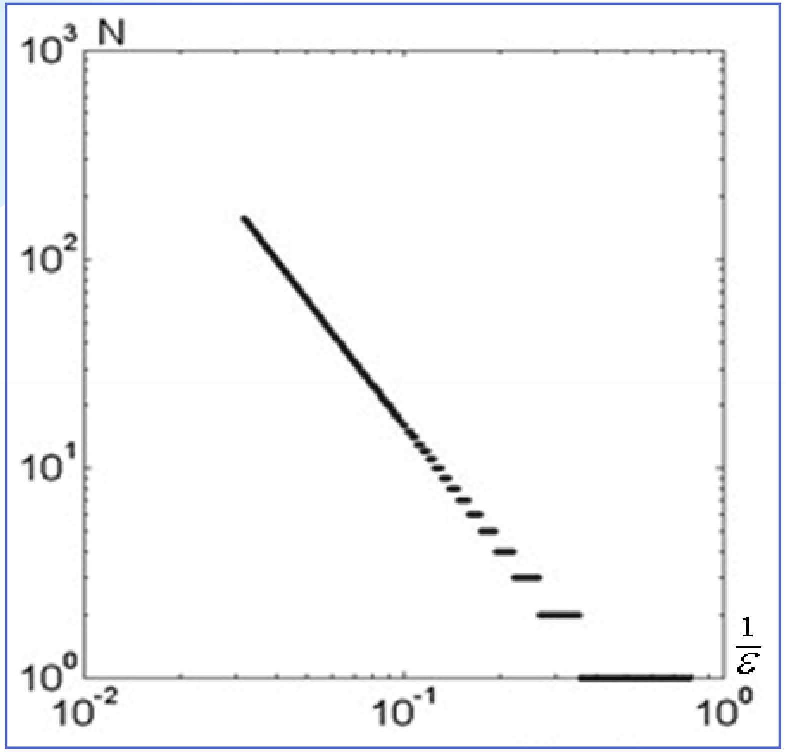
Dispersion Equation Solutions for points tanhx/x=-tany/y
ω
25
20
15
10
5
0
6
5
4
3
2
1
0
y for: tany/y
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x for: tanhx/x

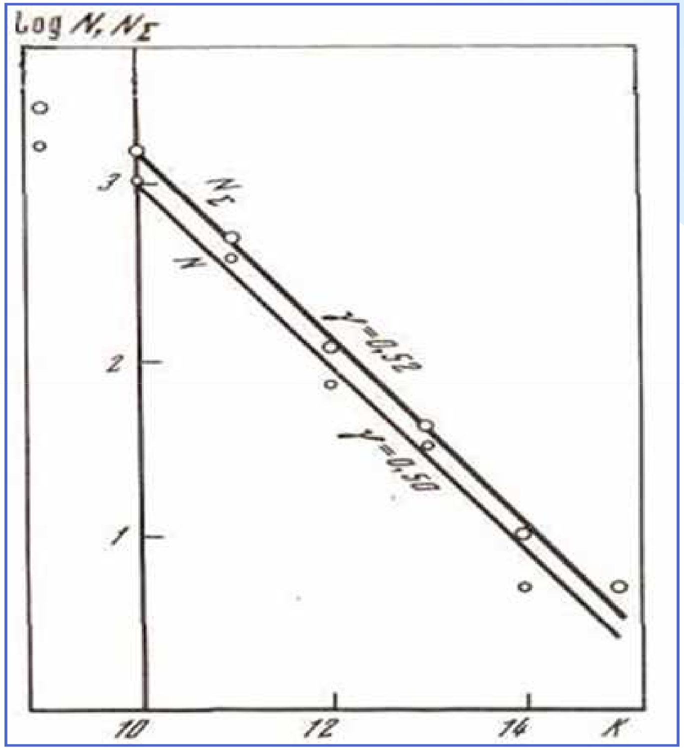
More large scale



Seismological law of Gutenberg-Richter

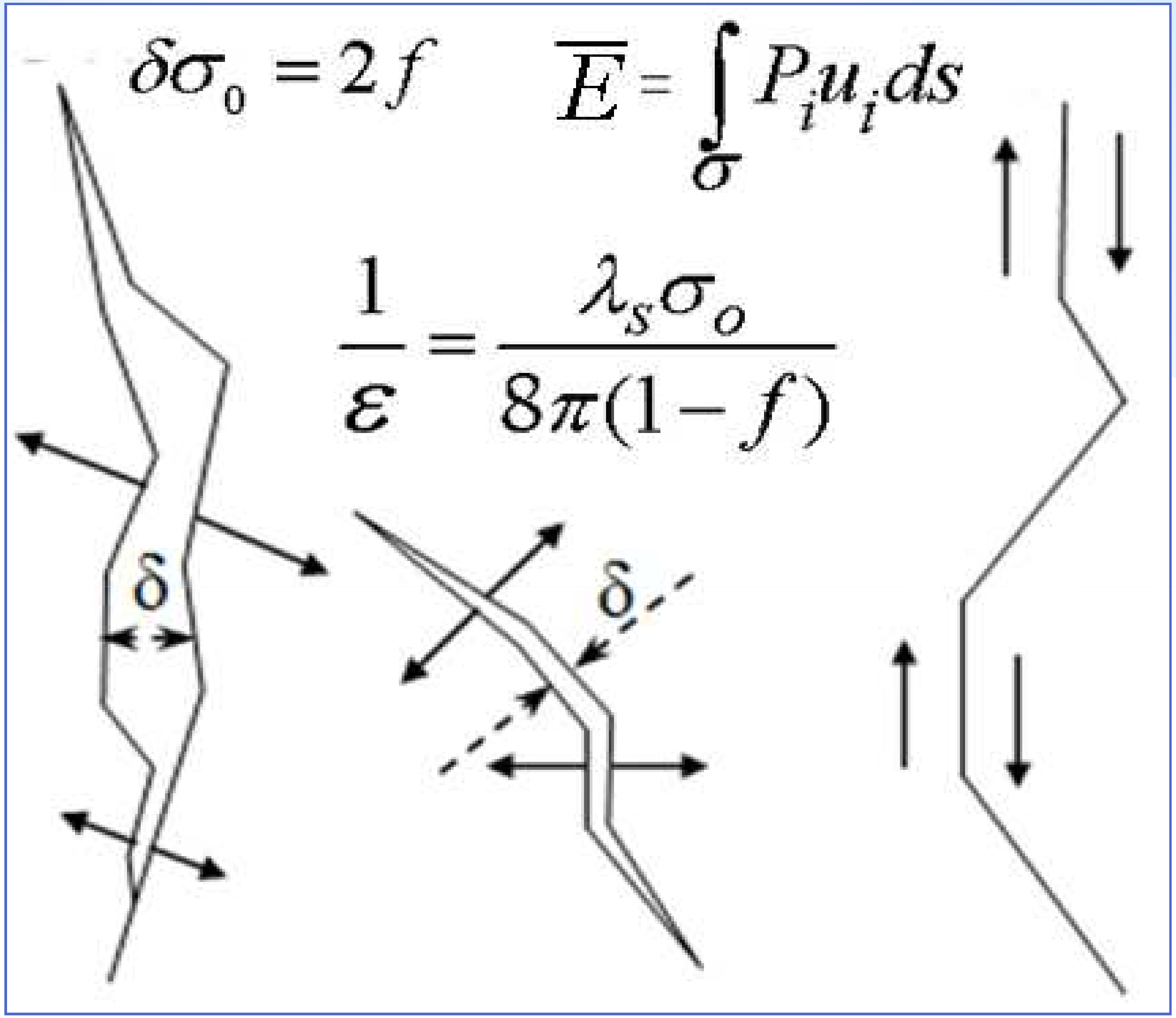
$$\frac{1}{\varepsilon} = \frac{\sigma_0 \lambda_S}{8\pi(1-f)}$$





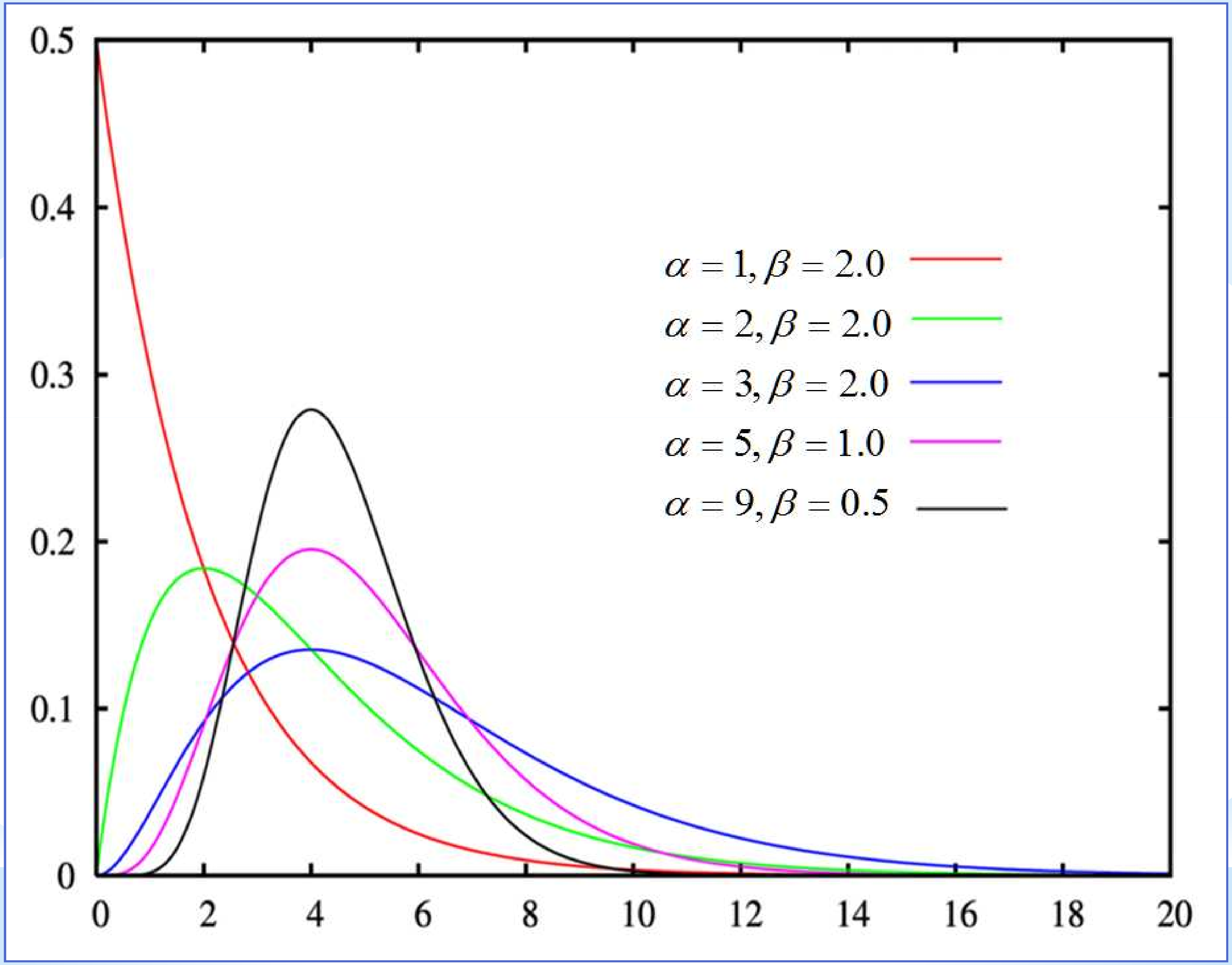
Left. Theoretical dependence of complex roots number versus cracks specific surface. Tangent of the angle γ = 0.5. It is clearly visible non-uniqueness of solutions with the great energies.

Right. Experimental dependence seismic events numbers versus energy, which is proportional to cracks specific surface. Tangent of the angle γ = 0.5 – 0.52



Real shear cracking process (right hand) without of sufficient crack opening corresponds to Richter-Gutenberg law

Gamma distribution of random value *l0*



Gamma-distribution of sizes of random structures.

$$f(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$$Mx = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \int_0^{\infty} x^{\alpha} e^{-\beta x} dx =$$

$$= \frac{\beta^{\alpha}}{\Gamma(\alpha)} \frac{\Gamma(\alpha+1)}{\beta^{\alpha+1}} = \frac{\alpha}{\beta}$$

$$\sigma^2 = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \int_0^{\infty} x^{\alpha-1} (x-1)^2 e^{-\beta x} dx = \frac{\alpha}{\beta^2} = \frac{1}{\alpha}$$

P-operator for random size of structure l_0

$$P(D_x, D_y, D_z, \alpha) = \frac{1}{2}\int_{-1}^{1}\left(\frac{\alpha}{\alpha - l_0\sqrt{\Delta}t}\right)^{\alpha} dt$$

Dispersion equation for random structures

$$\frac{1}{2}\int_{-1}^{1}\left(\frac{\alpha}{\alpha - ikl_0 t}\right)^{\alpha} dt = \frac{k_S^2}{k^2}$$

If $\alpha \to \infty$ $\quad \frac{\sin(kl_0)}{kl_0} = \frac{k_S^2}{k^2}$

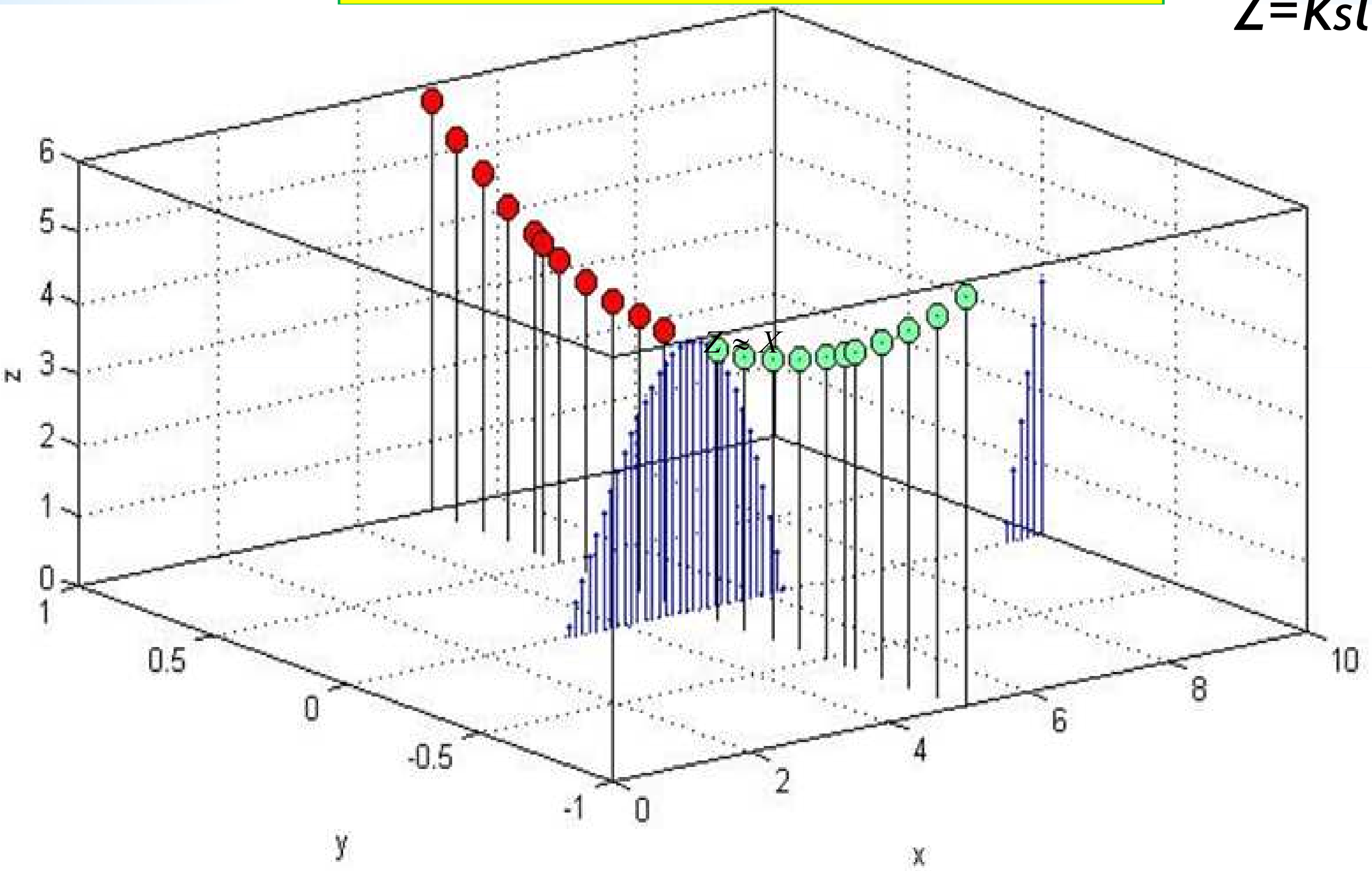
We returned to usual dispersion equation

$$P\frac{\partial \sigma_{ik}}{\partial x_k} = (P - E)\rho \ddot{u}_i$$

Equation of equilibrium at whole and equation of motion in micro-scale

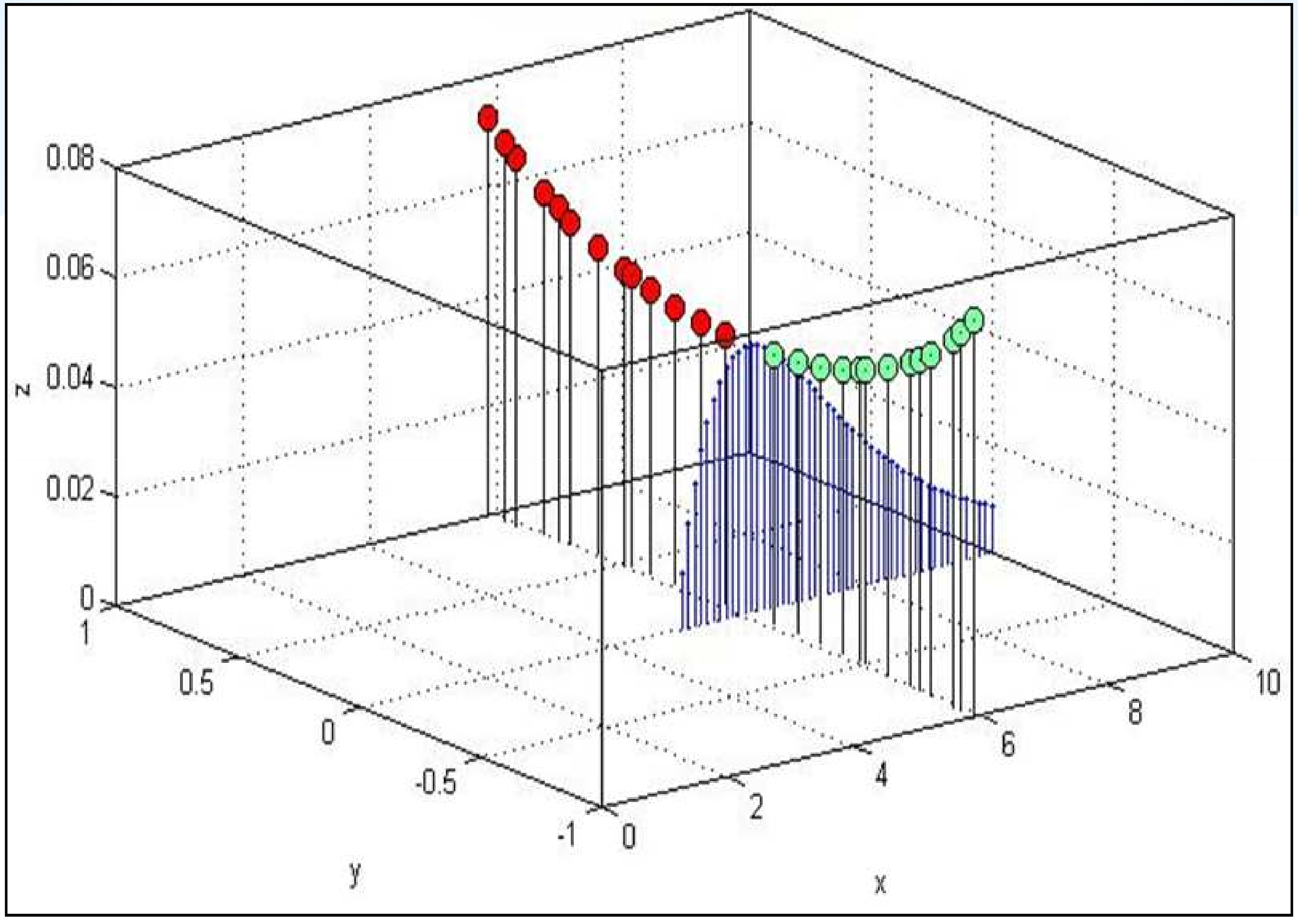
Waves due to statics at $\alpha = 300$
i.e. at small variance of blocks sizes (z=x+iy)

$X = kl_0$
$Z = k_s l_0$



Red color-fast catastrophes, green color-damping scenarios, blue points mean oscillations. Z~X means fast process, Z<<X means slow wave process.

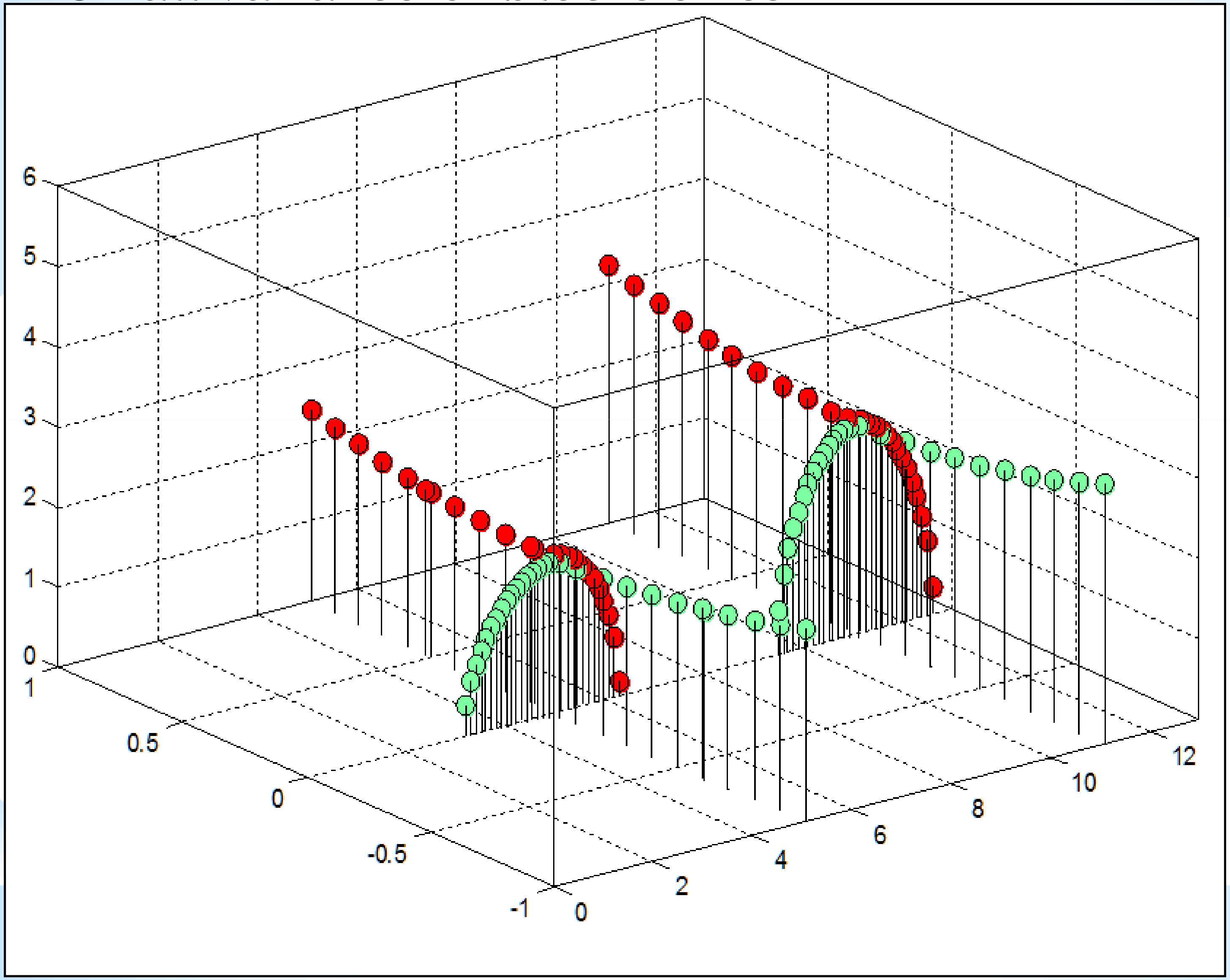
Waves due to statics at $\alpha = 5$
i.e. at large variance of blocks sizes



Red color-slow catastrophes, green color-damping scenarios, blue points mean oscillations. Z~X means fast process, Z<<X means slow wave process.

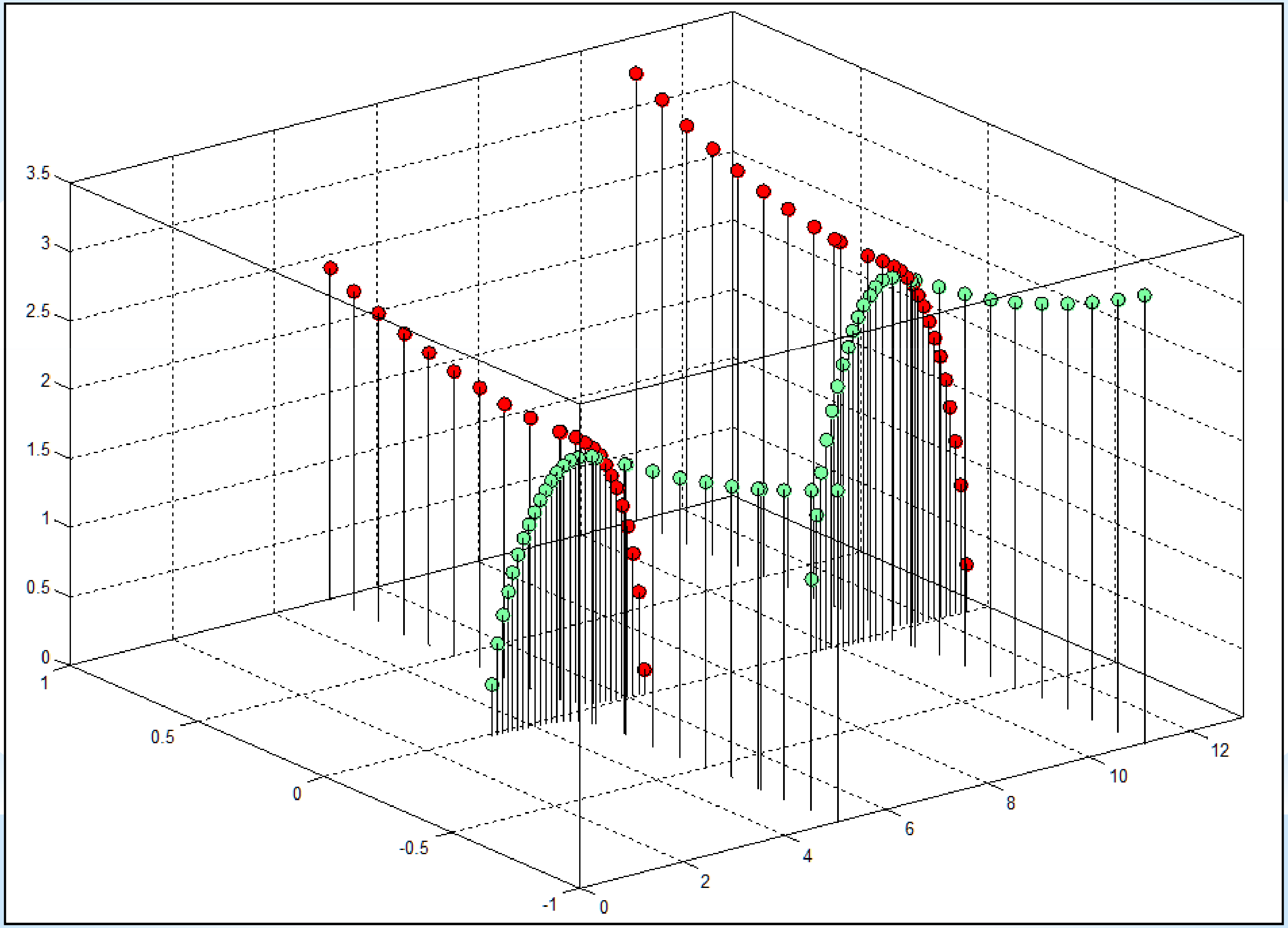
Waves due to statics at $\alpha = 300$ with friction $p\delta = 0.01$

Small variance of blocks sizes

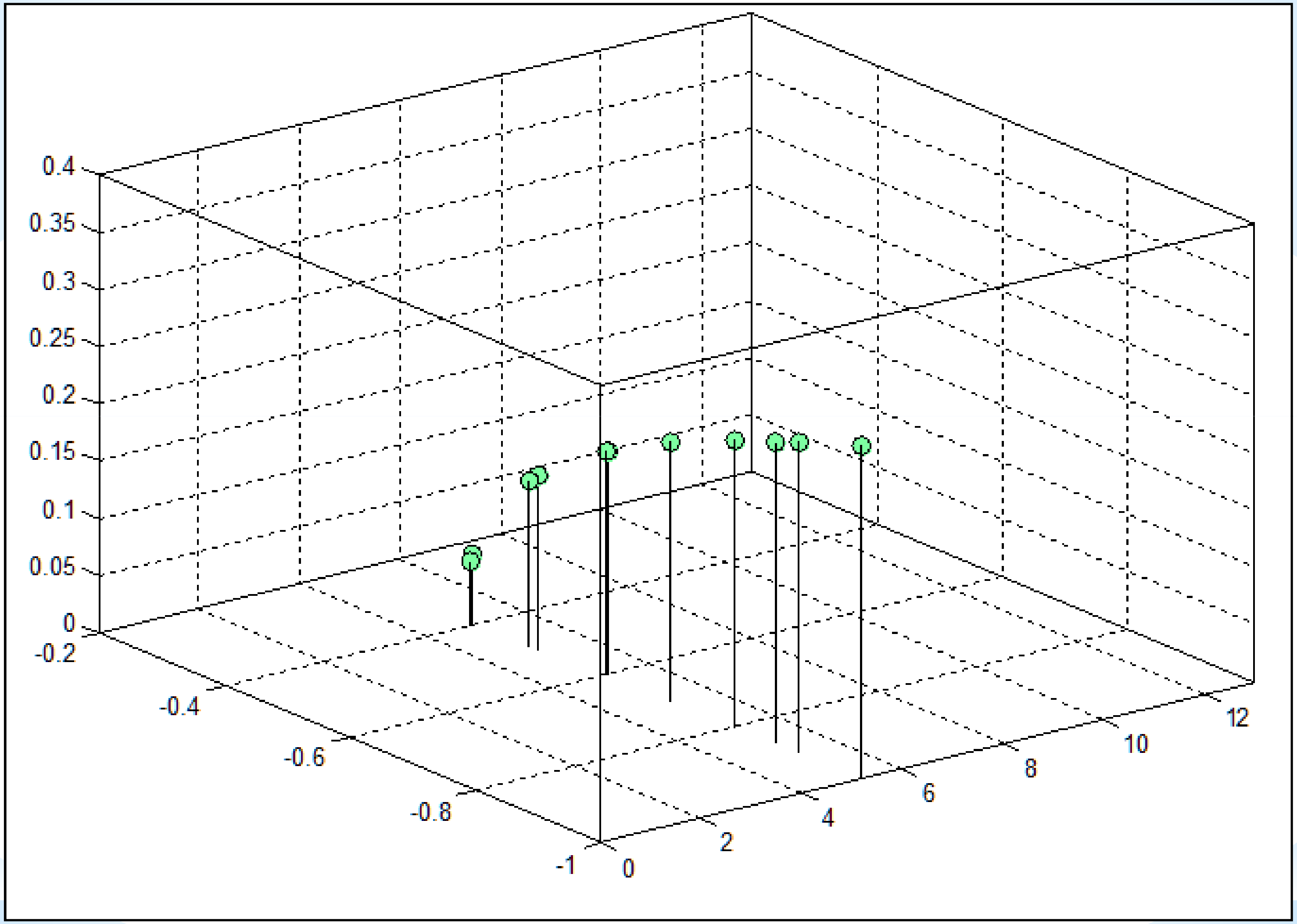


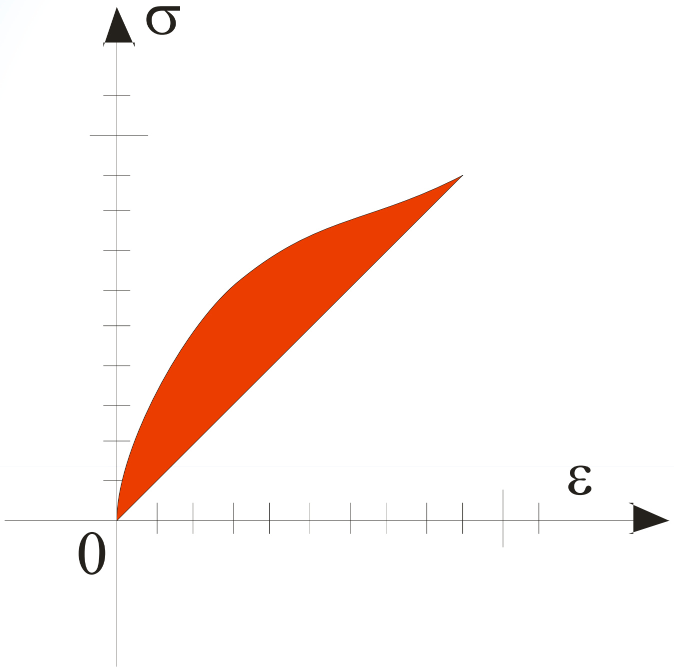
Waves due to statics at $\alpha=300$ with friction $p\delta=0.05$

Small variance of blocks sizes



Large variance of blocks sizes.





Stress-strain diagram. The nonlinear loading and linear unloading

Equation of motion in long wave approach

$$\frac{\partial}{\partial x_k}\left(E + \Delta\frac{l_0^2}{3!}\right)\sigma_{ik} = \rho\ddot{u}_i$$

$$u_{xx}(1 - 2b^2 u_x) + \frac{l_0^2}{3!}u_{xxxx} = \frac{1}{c^2}u_{tt}$$

$$\xi = t - x/t;\ \eta = t + x/t$$

$$\varphi_0\frac{\partial}{\partial\eta},\ u\ l_0\frac{\partial}{\partial\eta}$$

Are values of more small order

Weak nonlinearity and dispersion

$$u_\eta - b^2 u u_\xi + \frac{l_0^2}{3!}u_{\xi\xi\xi} = 0$$

Solution we are looking in the moving wave form

$$u = cTF\left(\frac{t - \alpha x/c}{T}\right), \qquad \alpha > 1$$

Assuming $F' = \varphi(\xi)$ We have nonlinear equation

$$\varphi'' + \frac{3!(\alpha^2 - 1)}{l_0^2 \alpha^4}\varphi = -\frac{3!b^2}{l_0^2 \alpha}\varphi^2$$

If $$\frac{3!(\alpha^2 - 1)(cT)^2}{l_0^2 \alpha^4} = 1, \quad \alpha = 1 + \frac{1}{2}\frac{l_0^2}{(cT)^2} = 1 + \frac{1}{2}\varepsilon^2 \; ; \; \varphi = \varphi_0 \overline{\varphi}$$

$$\varepsilon = \frac{l_0}{2cT} = \frac{l_0}{2\lambda_S}$$

There is this equation in more simple form

$$\varphi'' + \varphi + \beta\varphi_0\varphi^2 = C = 0, \quad \beta = \frac{6b^2}{\alpha\varepsilon^2}, \; \beta\varphi_0 - small\ parameter$$

It accurate solution is

$$\frac{t - \alpha x / c}{T} = \int_0^{\varphi} \frac{dp}{\sqrt{1 - p^2 - \beta\varphi_0 p^3}}$$

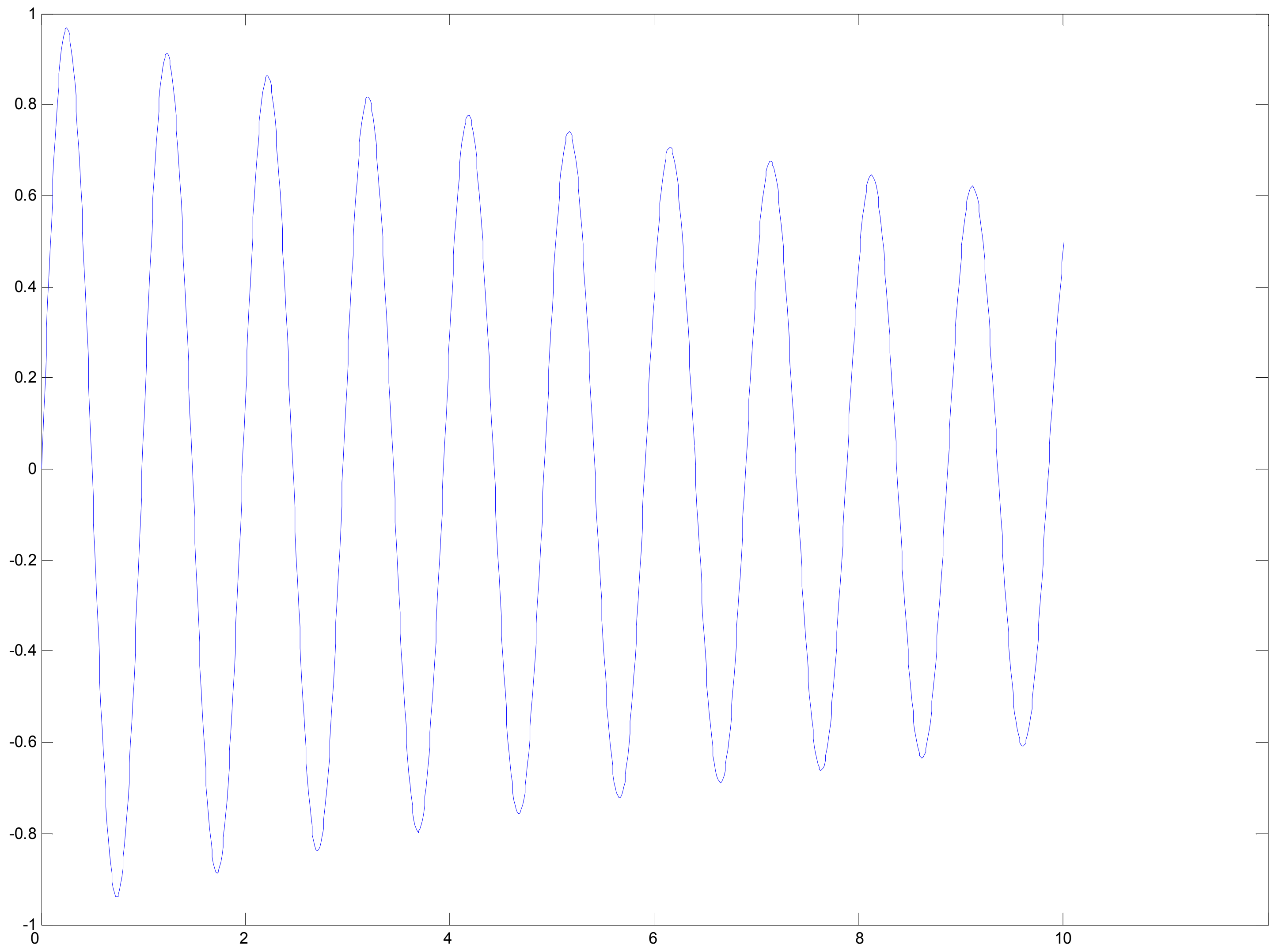
This solution tends in limit $\varphi_0 \to 0$

To usual sinusoidal curve, while a common solution of nonlinear equation describes more wide class of phenomena

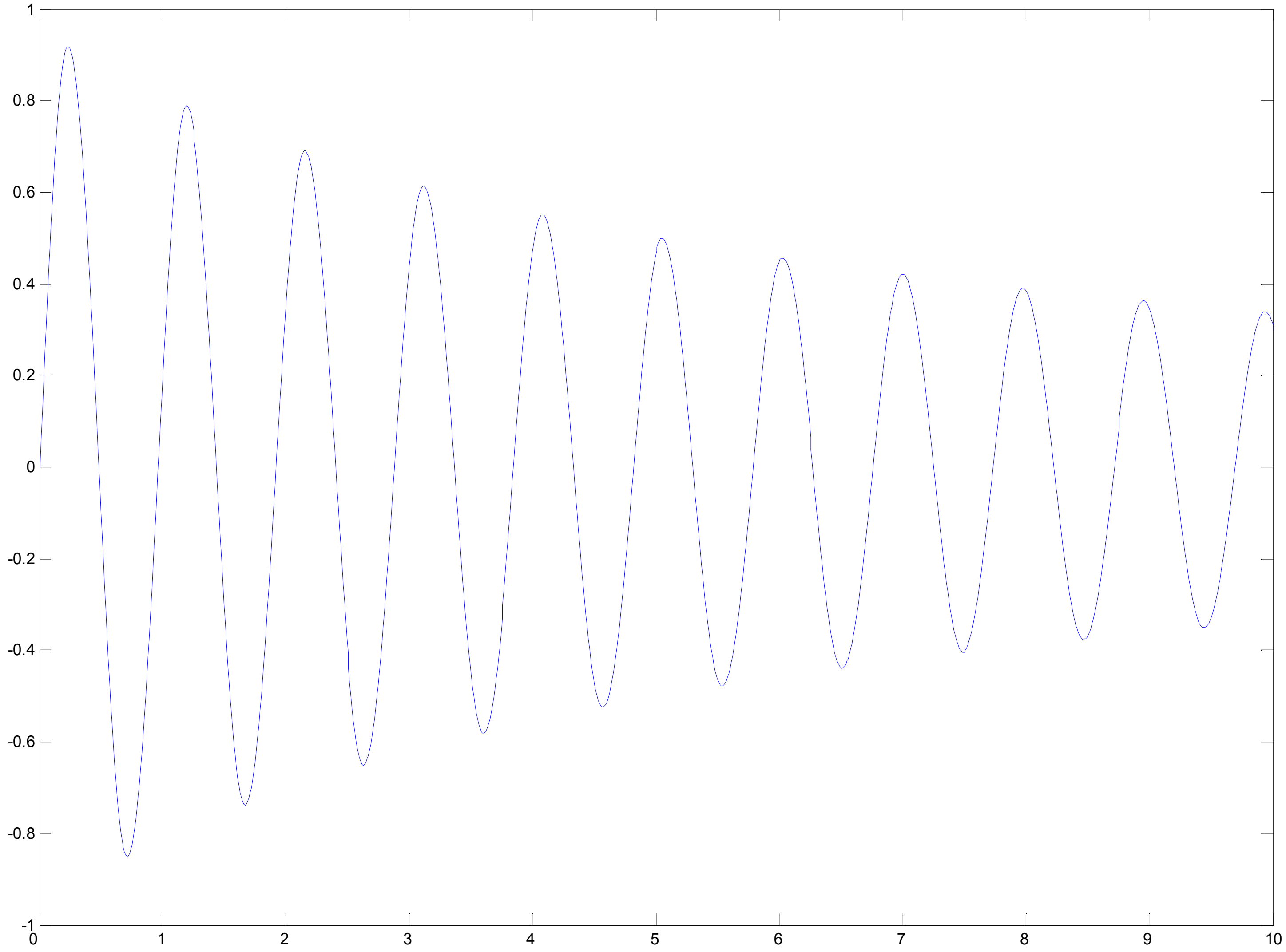
$$\frac{t-\alpha x/c}{T} = \int_0^{\varphi} \frac{dp}{\sqrt{C_1 + Cp - p^2 - \beta\varphi_0 p^3}}$$

The whole point is that in denominator there is very small value

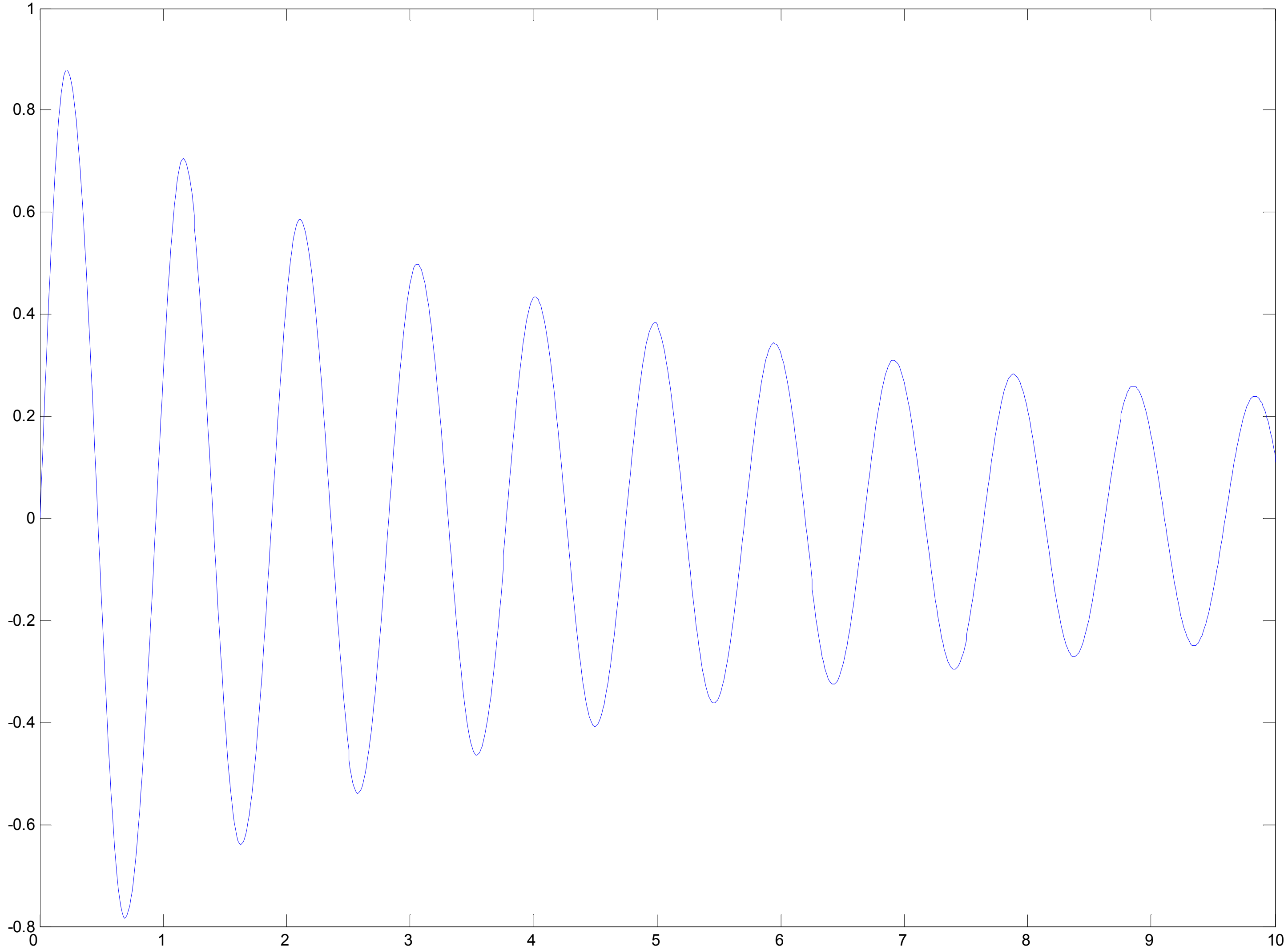
$$\frac{\varphi_0}{\varepsilon^2} = \frac{\varphi_0 (cT)^2}{l_0^2} >> \varphi_0$$



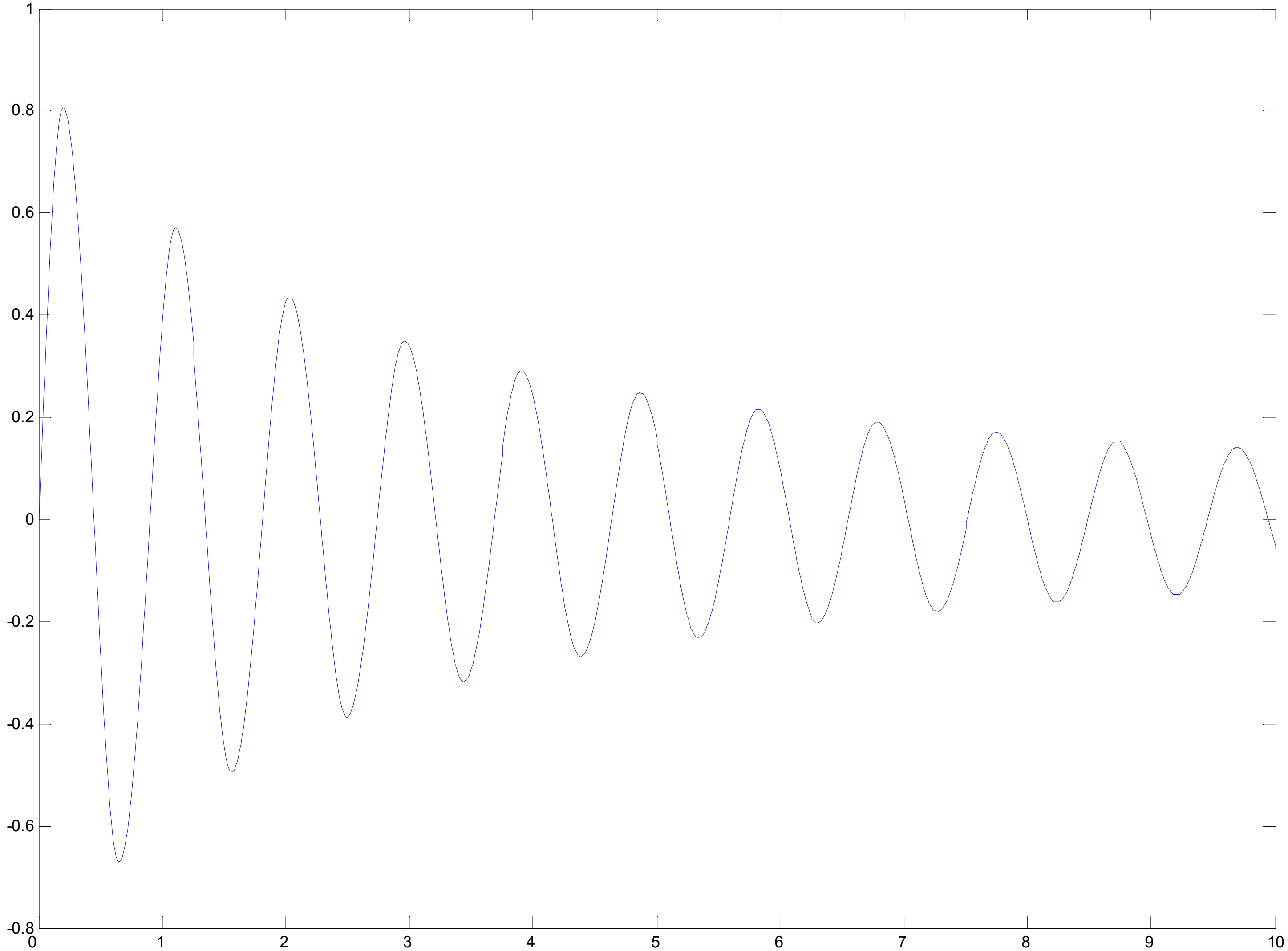
The attenuation with distance due to nonlinearity.
The nonlinear parameter 0.1



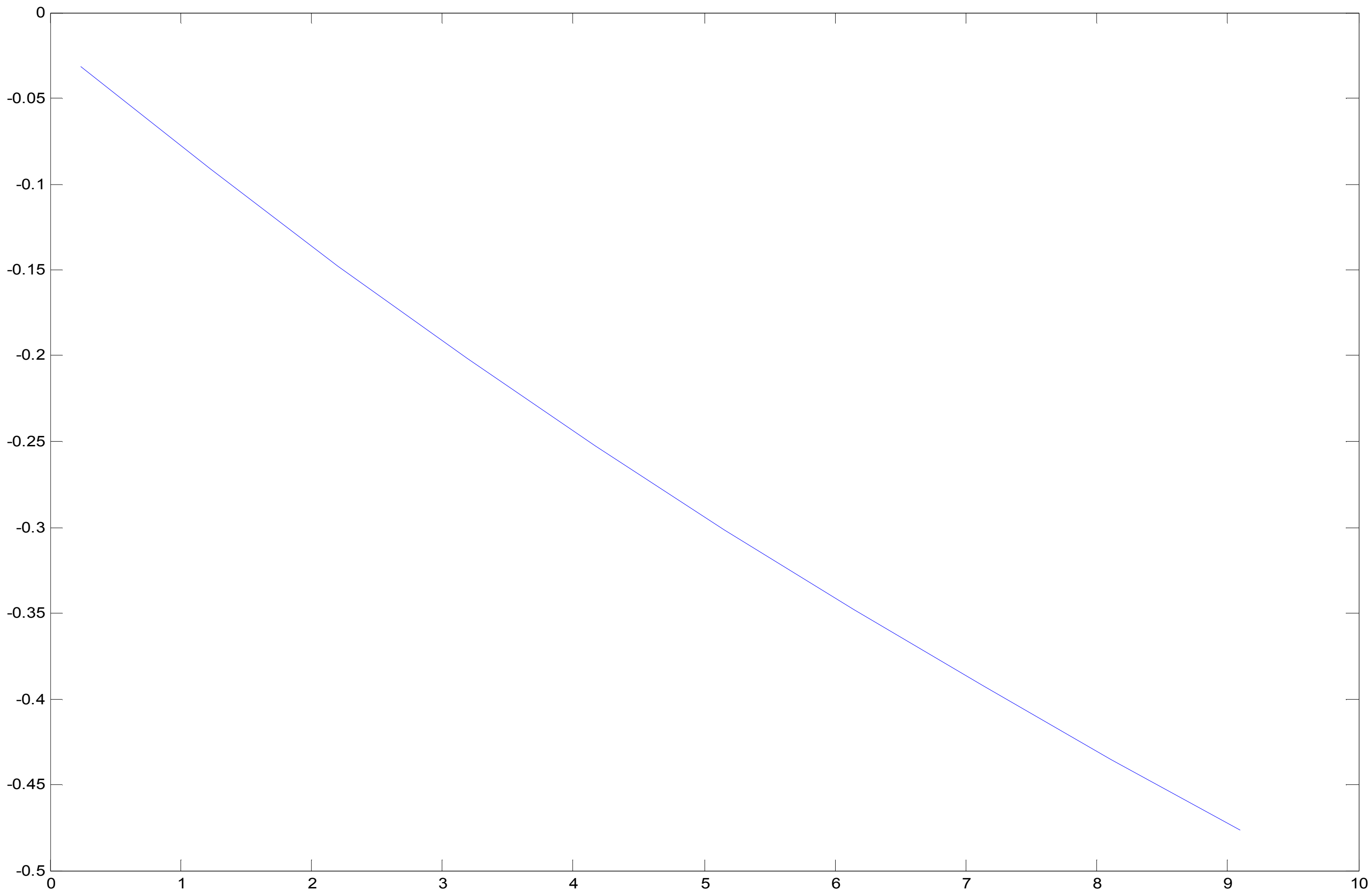
The nonlinear parameter 0.3



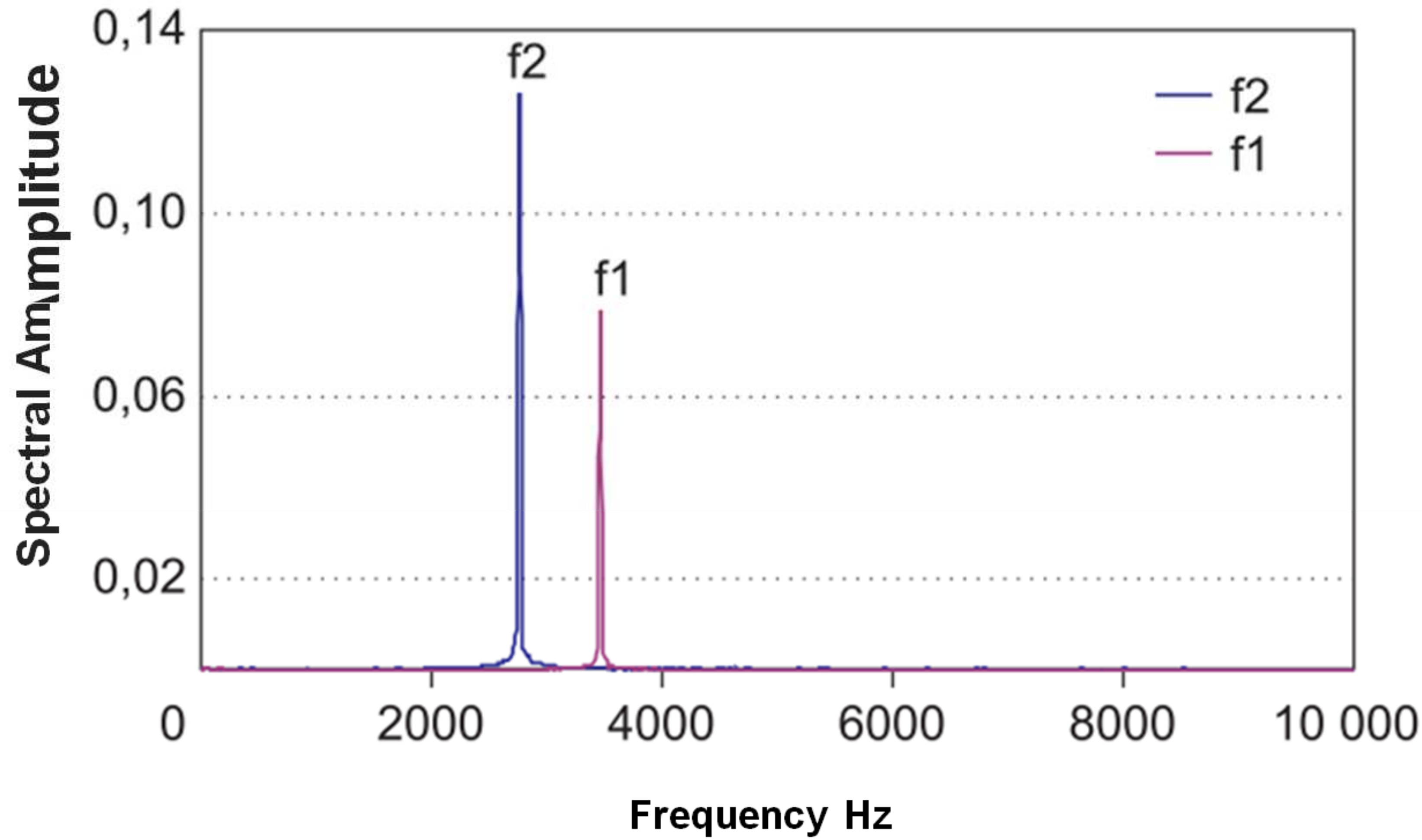
The nonlinear parameter 0.5



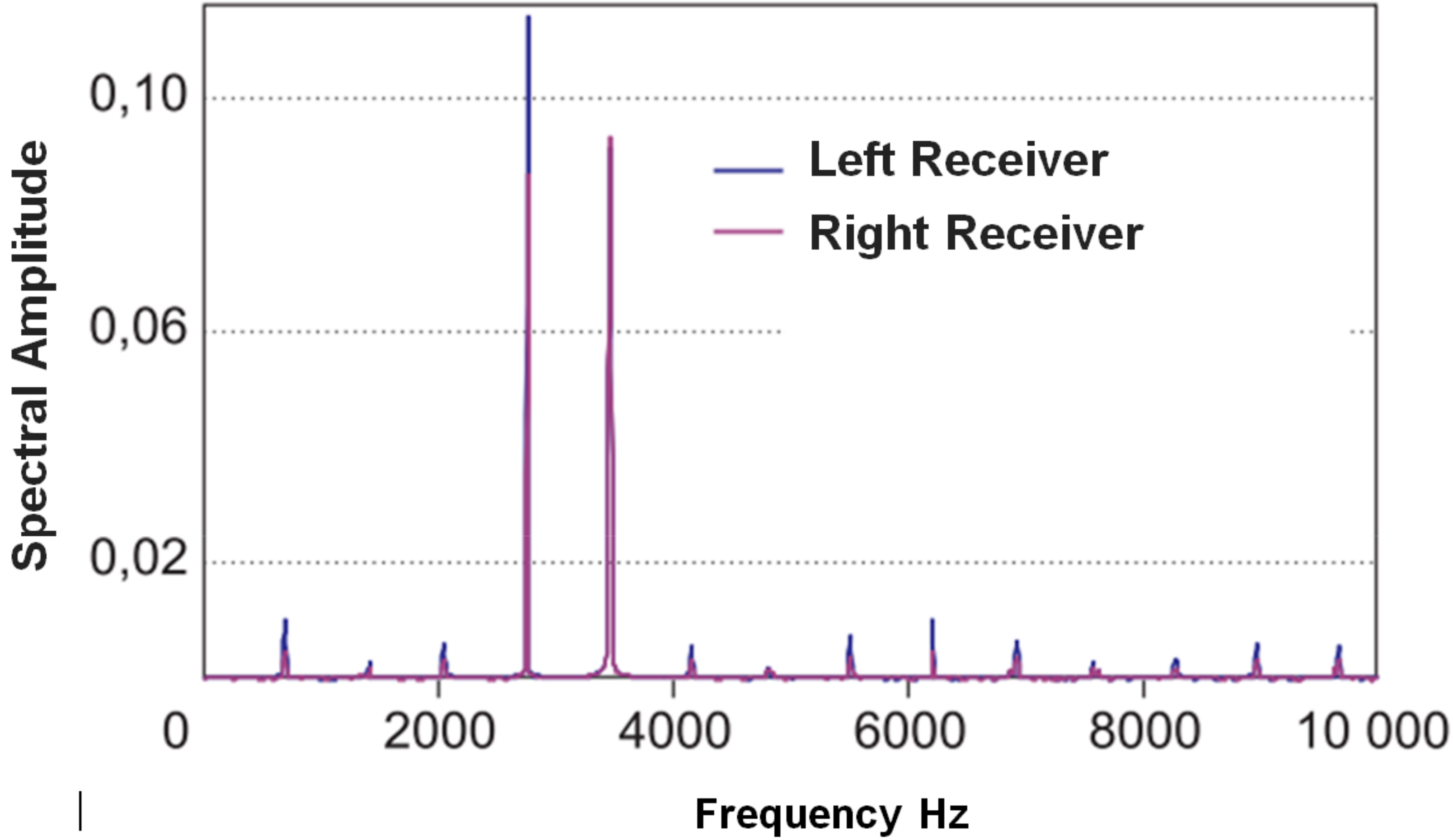
The nonlinear parameter 1



The amplitude logarithm at nonlinear parameter 0.1



Spectrum of the excitation of transverse waves at a frequency of f1 and f2 (G.V. Egorov, E.I. Mashinskii, 2011)



Spectrum of combined frequencies of shear wave on the left (a) and rihgt (b) sizes of specimen
(G.V. **Egorov**, E.I. Mashinskii, 2011)

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4. **Sibiryakov B.P., Prilous B.I., Kopeykin A.V.**
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Thank you for attention